Healthy extension of mimetic dark matter and its cosmology

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What’s DM origin?

Particle models

- **WIMP** (Weak Interacting Massive Particle)  
  Steigman, Turner (1985)

- **Axion DM**  
  Abbott, Sikivie (1983)

  sterile neutrino, PBH e.t.c.
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Does there exist particles beyond standard model?

.... hence there is no guarantee that DM origin are described by particle models.
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Does there exist particles beyond standard model?

…. however there is no guarantee that DM origin are described by particle model.

Is it possible to explain the DM origin by using alternative theories?
Alternatives of DM origin

Modified gravity models

- Horava-Lifshitz gravity  Mukohyama (2009), Horava, et al. (2011)

Bi-gravity, $f(R)$  e.t.c.
Alternatives of DM origin

Modified gravity models

- MOND, TeVeS \textit{\cite{Milgrom1983, Bekenstein2004}}
- Horava-Lifshitz gravity \textit{\cite{Mukohyama2009, Horava2011}}
- \textbf{Mimetic Dark Matter} \textit{\cite{Lim2010, Chamseddine2013}}

Bi-gravity, $f(R)$ e.t.c.

\textbf{MDM}

- a special class of \textit{scalar-tensor theory}
- It is possible to realize the same behavior as DM.

$$\rho_{\text{DM}} \propto \frac{1}{a^3}$$
Mimetic Dark Matter

Lim, et al. (2010), Chamseddine, Mukhanov (2013)

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} R - \lambda (g^\mu_\nu \partial_\mu \phi \partial_\nu \phi + 1) \right], \lambda: \text{Lagrange multiplier} \]

mimetic term

\[ \delta \lambda: \text{mimetic constraint} \quad g^\mu_\nu \partial_\mu \phi \partial_\nu \phi = -1 \quad (\phi: \text{non-dynamical}) \]

\[ \delta g: \quad G^\mu_\nu = T^\mu_{\text{DM} \nu}, \nabla_\mu T^\mu_{\text{DM} \nu} = 0 \]

dust-like: \quad T^\mu_{\text{DM} \nu} = \rho_{\text{DM}} u_\mu u_\nu, \quad \rho_{\text{DM}} = 2\lambda, \quad P_{\text{DM}} = 0

\[ u_\mu = \partial_\mu \phi \]
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origin of DM?
Extended MDM

Extension of MDM

\[
S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} R - \lambda (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 1) + \frac{\gamma}{2} (\Box \phi)^2 \right]
\]

\[
T_{DM\mu\nu} = 2\lambda \partial_\mu \phi \partial_\nu \phi + \gamma \left( \partial_\alpha \phi \partial^\alpha \Box \phi + \frac{1}{2} (\Box \phi)^2 \right) g_{\mu\nu} - \gamma (\partial_\nu \phi \partial_\mu \Box \phi + \partial_\mu \Box \phi \partial_\nu \phi)
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BG level: \( \rho_{DM} \propto \frac{1}{a^3} \), scalar dof is dynamical.
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Extension of MDM  Chamseddine, et al. (2013)

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\( \Rightarrow \) small scale behavior of perturbations can be improved.

Capela, Ramazanov (2013)

One may solve missing satellites, core-cusp...!
Extended MDM

Extension of MDM \hspace{1cm} \text{Chamseddine, et al. (2013)}

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But model has \textbf{gradient instability} at perturbation level.

\hspace{1cm} \text{Arroja, et al. (2015), Ijjas, Steinhadt (2016)}

Unstable
Extended MDM

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Unstable

IR limit of projectable HL

Mukohyama (2009)

imperfect
Our aim and goal

Stabilization of extended MDM?

One can consider healthy theory of MDM by using general framework of scalar-tensor theory;

**XG3 theory** (2014, Gao): largest class recently used in application for cosmology

⇒ stability conditions
Our aim and goal

Stabilization of extended MDM?

XG3 theory (2014, Gao) \implies \text{stability conditions}

Our aim; unifying manner of s-t theory

- By observations one can constrain any concrete models in model’s class at one time.
- If one can extend endlessly one’s model, can one find the model which has better feature?

\implies \text{Realized models would have special property, so we would like to find these minimal healthy model as DM.}
Our aim and goal

Stabilization of extended MDM?

XG3 theory (2014, Gao) \(\Rightarrow\) stability conditions

Our aim; unifying manner of s-t theory

- By observations one can constrain any concrete models in model’s class at one time.

- If one can extend *endlessly* one’s model, can one find the model which has *better feature*?

\(\Rightarrow\) Realized models would have *special property*, so we would like to *find its minimal healthy model and specialities*.  

Our goal

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**XG³** *(eXtended Galileon with 3 space)*  

**Spatially covariant theory** (having only one scalar d.o.f.)

\[ \mathcal{L} = \mathcal{L} (K_{ij}, R_{ij}, D_i, a_i) \]  
\[ \text{in unitary gauge } \phi = t \]

3-dim. geometrical quantities

**Simplifying assumptions**

- Up to **second order** in derivatives in covariant Lagrangian
  \[ \nabla^2 \phi, \nabla_\mu \nabla_\nu \phi \]

- Second-order terms are limited up to be **cubic**
  \[ (\nabla^2 \phi)^3, (\nabla_\mu \nabla_\nu \phi)^3 \]
$XG_3$ (eXtended Galileon with 3 space)  
Gao (2014)

**XG3 theory** arbitrary funcs.; 28 terms

\[
\frac{1}{\sqrt{-g}} \mathcal{L}_{XG3} = \mathcal{K}_1 + \mathcal{K}_2 + \mathcal{K}_3 + \nu
\]

\[
\mathcal{K}_1 = (a_0 + a_1 R + a_3 R^2 + a_4 R_{ij} R^{ij} + a_5 a_i a^i) K
\]
\[
+ [ (a_2 + a_6 R) R^{ij} + a_7 R^i_k R^{kj} + a_8 a^i a^j ] K_{ij}
\]

\[
\mathcal{K}_2 = (b_1 + b_3 R) K^2 + (b_2 + b_4 R) K_{ij} K^{ij} + (b_5 K K_{ij} + b_6 K_{ik} K^k_j) R^{ij}
\]

\[
\mathcal{K}_3 = c_1 K^3 + c_2 K K_{ij} K^{ij} + c_3 K^i_k K^j_k K^k_i
\]

\[
\nu = d_0 + d_1 R + d_2 R^2 + d_3 R_{ij} R^{ij} + d_4 a_i a^i + d_5 R^3 + d_6 R R_{ij} R^{ij}
\]
\[
+ d_7 R^i_j R^{j_k} R^k_i + d_8 R a_i a^i + d_9 R_{ij} a^i a^j
\]

08/13
Mimetic XG3

Hirano, Nishi, Kobayashi
in preparation

Lagrangian in unitary gauge \( \phi = t \)

\[
\mathcal{L}_{\text{MXG3}} = \mathcal{L}_{\text{XG3}} - N \sqrt{\gamma} \lambda \left( -\frac{1}{N^2} + 1 \right)
\]

Stability of flat FLRW background?

Stability at perturbation level \( \Rightarrow \) necessary to study both scalar and tensor parts
**Tensor part**  

**Metric**  
\[ ds^2 = -dt^2 + a(t)^2 \left( \delta_{ij} + h_{ij} + \frac{1}{2} h_k^i h_k^j \right) \]

\( N = 1 \); mimetic term does not contribute to\(^{(2)} S\).

\[ \Rightarrow \text{Tensor perturbation does not change from XG3 case.} \]

**Quadratic action**

\[ (2) S_T = \frac{1}{8} \int dt d^3x \ a^3 \left[ G_T \dot{h}_{ij}^2 - \frac{F_T}{a^2} (\partial h)^2 - \frac{W_T}{a^4} (\partial^2 h)^2 \right] \]

\[ G_T = 2[b_2 + 3(c_2 + c_3)H], \quad W_T = -2[d_3 + (3a_4 + a_7)H] \]

\[ F_T = 2[d_1 + (3a_1 + a_2)H + (9b_3 + 3b_4 + 3b_5 + b_6)H^2] + \frac{1}{a} \frac{d}{dt} [a \{a_2 + (3b_5 + 2b_6)H\}] \]

**stability conditions for tensor:**  
\[ G_T > 0, \quad F_T > 0, \quad W_T > 0 \]
**Scalar part**  

**Metric**  
\[ ds^2 = -N^2 dt^2 + \gamma_{ij} (dx^i + N^i dt) (dx^j + N^j dt) \]
\[ N = 1 + \alpha, \ N_i = \partial_i \beta, \ \gamma_{ij} = a^2 e^{2\xi} \delta_{ij} \]

**Mimetic term:**  
\[ N \sqrt{\gamma} \lambda \left( -\frac{1}{N^2} + 1 \right), \ \lambda = \bar{\lambda} + \delta \lambda \]
\[ \delta \lambda : \ \alpha = 0 \quad \Rightarrow \quad \text{EoM of} \ \alpha \text{ vanishes.} \]
\[ \beta \text{ constraint equation only} \]

**Quadratic action**  
Hirano, Nishi, Kobayashi  in preparation

\[ (^{(2)}S_\zeta) = \int dt d^3 x \ a^3 \left[ G_S \dot{\zeta}^2 - \frac{F_S}{a^2} (\partial \zeta)^2 - \frac{W_S}{a^4} (\partial^2 \zeta)^2 \right] \]

**stability conditions for scalar:**  
\[ G_S > 0, \ F_S > 0, \ W_S > 0 \]
Minimal extension model

\[ \mathcal{L}_{\text{min}} = \mathcal{L}_{\text{eMDM}} + \mathcal{L}_{\text{new}} \]

\[ \mathcal{L}_{\text{eMDM}} = \frac{1}{2} (R + K_{ij} K^{ij} - K^2) + \lambda \left( 1 - \frac{1}{N^2} \right) + \gamma K^2 \]

\[ \mathcal{L}_{\text{new}} = a_1 K R + d_2 R^2 \]

check Background

\[ \frac{3(2 - 3\gamma)}{2} H^2 = \rho_{\text{DM}} + \rho \quad \text{regular} \quad \Rightarrow \quad 2 - 3\gamma > 0 \quad (\gamma < 2/3) \]

\[ -(2 - 3\gamma) \dot{H} = \rho_{\text{DM}} + \rho + p \]

\[ \rho_{\text{DM}} \propto \frac{1}{a^3} \quad \text{(in every epoch)} \]

\[ \rho, \ p : \text{energy density and pressure of standard matter} \]
Minimal extension model

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\[ \mathcal{L}_{\text{new}} = a_1 K R + d_2 R^2 \]

**Check** Perturbation (BG: \( \gamma < 2/3 \))

\[ \mathcal{G}_T = 1 > 0 \ , \ \mathcal{F}_T = 1 + 6a_1 H > 0 \ , \ \mathcal{W}_T = 0 \]

\[ \mathcal{G}_S = -\frac{2 - 3\gamma}{\gamma} > 0 \quad \Rightarrow \quad \gamma < 0 \ , \ \mathcal{W}_S = \frac{4a_1^2}{\gamma} - 16d_2 \geq 0 \]

\[ \mathcal{F}_s = \frac{2 - 3\gamma}{\gamma} \left( 2a_1 H + 2\dot{a}_1 \right) + 6a_1 - 1 > 0 \]

- Lagrangian must have \( R^2 \) terms.
- \( k^4 \) terms can be eliminated.
Summary

Mimetic Dark Matter

• Non-dynamical scalar d.o.f. ⇒ DM(dust solution)
• Adding higher derivative
  ⇒ imperfect form, scalar d.o.f. is dynamical

Mimetic XG3

• Healthy MDM theory without any instabilities by using XG3 theory
• Minimal models include two additional terms.

Future direction

gravitational test in solar system, cosmological evolution, rotation curve, BH…