Dynamical angled brane

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[arXiv:1603.01948 [hep-th]]
Introduction

The key word of this talk is “Dp-brane”.

The name “Dp-brane” is generally used to indicate a classical solution which is extended in $p$ directions.

Then, it has $p$ spacelike translational Killing vectors in the context of a theory containing gravity.
What is $p$-brane?


★ Classical membrane solution of Einstein equation

$X^1, X^2, \ldots, X^p$

- This is extended in $p$ direction.
- $p$-brane has $p$ spacelike translational Killing vectors.
\star p-(mem)brane

Ex)
0-brane: black holes, magnetic monopoles
... These are prototypes of p-branes for the case p=0.

\textit{p-brane configurations are essentially in view of the role they play in the supergravity or string theory.}
String theory, supergravity theory:

There are anti-symmetric tensor fields of higher rank.

$(p+1)$-form gauge field in $D$-dimensions:

There is $(p+2)$-form field strength.

$\Rightarrow$ A charged objects $(p$-dim) couples to $(p+1)$-form gauge field.
These are often called "dynamical p-brane" when the background spacetime describes the expanding (or contracting) universe.

These involves not only charged object in 4-dimensions but also M-branes in 11 dimensions and D-branes in ten dimensions.
Time dependent solutions in string theory:

- **D$^3$-brane in the IIB String**
  (Gibbons, Lu, Pope, Phys.Rev.Lett.94:131602,2005)
  (Kodama; Uzawa, JHEP 0507:061,2005)

- **Intersecting brane backgrounds**
  (Binetruy, Sasaki, Uzawa, Phys.Rev.D80:026001,2009)
  (Maeda, Ohla, Uzawa, JHEP 0906:051,2009)

- **Probe branes**

- **D-brane with angles**
  (Maeda, Uzawa, arXiv:1603.01948 [hep-th])

The branes are parallel or orthogonal.

Different branes have embeddings with angles.
The purpose of the work is …

• Analysis of brane dynamics by use of exact solutions in higher dimensions

(i) Construction of new time-dependent solution in string theory

(ii) Their applications to dynamics of branes
Today I’d like to tell you about …

- construct of dynamical brane with angles.

- the singularity in the geometry.

- the brane dynamics such as brane collision.
**Dp-brane with angles**


Any number of Dp-branes whose relative orientations are given by certain SU(2) rotations.
Dynamical N Dp-brane with angles
(Maeda & Uzawa, arXiv:1603.01948 [hep-th])

\[ ds^2 = h^{p-7 \over 8} (t, z) \left[ -dt^2 + \gamma_{ij} dy^i dy^j \right] + \delta_{mn} d\zeta^m d\zeta^n + h(t, z) \delta_{ab} dz^a dz^b \]

\[ \gamma_{ij} dy^i dy^j = \delta_{ij} dy^i dy^j \]

\[ + \sum_{\alpha=1}^{N} f(\alpha) \left[ \left\{ (R(\alpha))_i^1 dy^i \right\}^2 + \left\{ (R(\alpha))_j^3 dy^j \right\}^2 \right] \]
Behavior of the solutions

- The metric becomes singular at $h(t, z) = 0$.

- The spacetime is regular when $h(t, z) > 0$.

$$h(t, z) = 1 + \sum_{\alpha=1}^{N} f_{(\alpha)} + \sum_{\alpha<\beta}^{N} f_{(\alpha)} f_{(\beta)} \sin^2 (\theta_{\alpha} - \theta_{\beta}),$$

$$f_{(\alpha)}(t, z) = \tilde{f}_{(\alpha)}(t) + \bar{f}_{(\alpha)}(z),$$

$$\tilde{f}_{(\alpha)}(t) = c_{(\alpha)0} t + c_{(\alpha)1}, \quad \bar{f}_{(\alpha)}(z) = \bar{c}_{\alpha} + \frac{Q_{\alpha}}{|z^a - \bar{z}_{(\alpha)}^a|^{5-p-d}},$$

$d_s$: number of the smeared directions

Spacetime cannot be extended beyond this region because the spacetime evolves into a curvature singularity.
Collision of two D3-branes:

The proper distance given in D3-brane is depicted. 3-dim transverse spaces are smeared.

(a) If two D3-branes satisfy $Q_1=Q_2$, it causes the complete collision at $\theta=0, \pi, 2\pi$ simultaneously.

(b) For $Q_1 \neq Q_2$, a singularity appears at $t=t_s>0$ when the distance is still finite.
The proper distance given in D3-brane is depicted. 1-dim transverse space is smeared.

Two D3-branes satisfy \( Q_1 = Q_2 \).

The proper distance is still finite when a singularity appears at \( t=1 \) on the brane located at \( z=0 \).
Comments

Dynamical Dp-brane with angles in the 10d String

- The solution describes any number of Dp-brane whose relative orientations are given by SU(2) rotations.

- In the far region from angled Dp-brane, the metric depends only on time.

- Angled D3-brane with smearing branes:

  ⇒ Colliding branes with same charges, and the orientation of configurations are either 0 or π.

  Otherwise, singularity appears before branes collide.