The late-time cosmic acceleration is discovered in 1998 and the source for this acceleration is named dark energy (DE).

Absence of conical singularities in beyond-generalized Proca theories

In scalar-tensor theories, it is generally difficult to realize weak gravity without any instabilities. However, in vector-tensor theories, generalized Proca (GP) theories can realize weak gravity healthily. De Felice, Heisenberg, Kase, Mukohyama, Tsujikawa and Zhang, PRD (2015)

\[ S_{\text{Gen-Proca}} = \int d^4x \sqrt{-g} \sum_{i=0} L_i, \]

\[ L_2 = G_2(X,F,Y), \quad L_3 = G_3(X)\nabla_\mu F^\mu, \quad L_4 = G_4(X) + G_{4,X}(\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla_\nu A^\mu, \)

\[ L_5 = G_5(X,G_\mu\nabla_\mu A^\nu) - \frac{1}{2} G_6(X) \nabla_\mu A^\nu \nabla_\mu \nabla_\nu A^\rho + 2 \nabla_\mu A_\nu \nabla_\rho A^\mu A^\nu - G_7(X) \nabla_\mu A_\nu \nabla_\rho A_\nu, \]

\[ L_6 = G_6(X) \nabla_\mu A_\nu \nabla_\rho A_\nu + \frac{1}{2} G_6(X) \nabla_\mu A_\nu \nabla_\rho A_\nu, \]

One cannot introduce Galileon-like interactions while keeping the U(1) gauge invariance (Deffayet et al. (2014)). GP theories are constructed by dropping the U(1) gauge invariance, i.e., Proca Lagrangian+Galileon-like interactions=GP theories [Heisenberg (2014)]. They have 2nd order EOMs and recover shift-symmetric Horndeski theories in the scalar limit.

Beyond-generalized Proca theories

GP Lagrangians are made of the Galileon-like functional forms of BGP theories. The growth rate of matter perturbations+Planck data tends to favor a lower growth rate than that predicted by GR. If the gravitational coupling becomes smaller, the lower growth rate can be realized.

The cosmological constant has a mild tension with the recent data.